ELEMENTARY GEOMETRY

FOR COLLEGE STUDENTS

6E

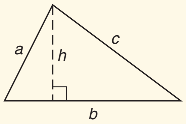
ALEXANDER ♦ KOEBERLEIN

Formulas

PLANE FIGURES:

P = Perimeter; C = Circumference; A = Area

Triangle:



$P = a + b + c$

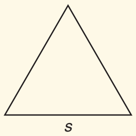
$A = \frac{1}{2}bh$

$A = \sqrt{s(s - a)(s - b)(s - c)},$

where s = semiperimeter, so

$s = \frac{1}{2}(a + b + c)$

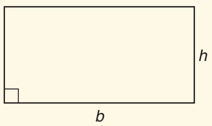
Equilateral Triangle:



$P = 3s$

$A = \frac{s^2}{4}\sqrt{3}$

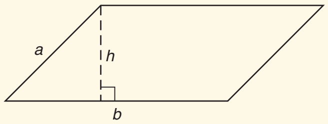
Rectangle:



$P = 2b + 2h$

$A = bh$ or $A = \ell w$

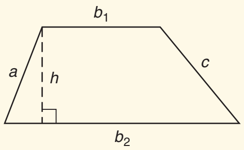
Parallelogram:



$P = 2a + 2b$

$A = bh$

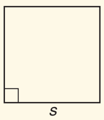
Trapezoid:



$P = a + b_1 + c + b_2$

$A = \frac{1}{2}h(b_1 + b_2)$

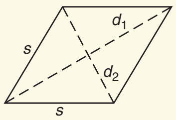
Square:



$P = 4s$

$A = s^2$

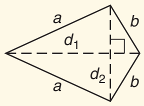
Rhombus:



$P = 4s$

$A = \frac{1}{2} \cdot d_1 \cdot d_2$

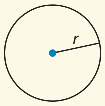
Kite:



$P = 2a + 2b$

$A = \frac{1}{2} \cdot d_1 \cdot d_2$

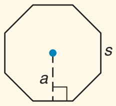
Circle:



$C = 2\pi r$ or $C = \pi d$

$A = \pi r^2$

Regular Polygon (n sides):

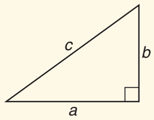


$P = n \cdot s$

$A = \frac{1}{2}aP$

MISCELLANEOUS FORMULAS:

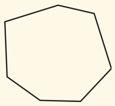
Right Triangle:



$c^2 = a^2 + b^2$

$A = \frac{1}{2}ab$

Polygons (n sides):

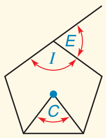


Sum (interior angles) $= (n - 2) \cdot 180°$

Sum (exterior angles) $= 360°$

Number (of diagonals) $= \frac{n(n - 3)}{2}$

Regular Polygon (n sides): I = Interior angle measure, E = Exterior angle measure, and C = Central angle measure

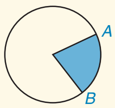


$I = \frac{(n - 2) \cdot 180°}{n}$

$E = \frac{360°}{n}$

$C = \frac{360°}{n}$

Sector:



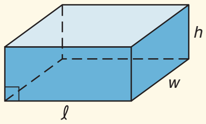
$\ell \widehat{AB} = \frac{m\widehat{AB}}{360°} \cdot 2\pi r$

$A = \frac{m\widehat{AB}}{360°} \cdot \pi r^2$

SOLIDS (SPACE FIGURES):

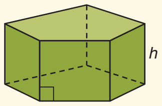
L = Lateral Area; T (or S) = Total (Surface) Area; V = Volume

Parallelepiped (box):



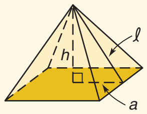
$T = 2\ell w + 2\ell h + 2wh$
$V = \ell wh$

Right Prism:



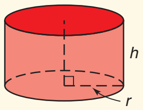
$L = hP$
$T = L + 2B$
$V = Bh$

Regular Pyramid:



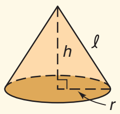
$L = \frac{1}{2}\ell P$
$\ell^2 = a^2 + h^2$
$T = L + B$
$V = \frac{1}{3}Bh$

Right Circular Cylinder:



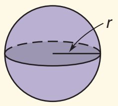
$L = 2\pi rh$
$T = 2\pi rh + 2\pi r^2$
$V = \pi r^2 h$

Right Circular Cone:



$L = \pi r\ell$
$\ell^2 = r^2 + h^2$
$T = \pi r\ell + \pi r^2$
$V = \frac{1}{3}\pi r^2 h$

Sphere:

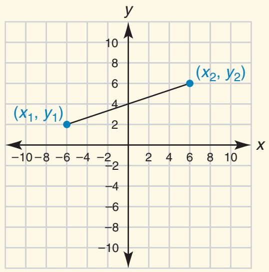


$S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Miscellaneous:
Euler's Equation: $V + F = E + 2$

ANALYTIC GEOMETRY:

Cartesian Plane



Distance:
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint:
$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}, x_1 \neq x_2$

Parallel Lines:
$\ell_1 \parallel \ell_2 \leftrightarrow m_1 = m_2$

Perpendicular Lines:
$\ell_1 \perp \ell_2 \leftrightarrow m_1 \cdot m_2 = -1$

Equations of a Line:
Slope-Intercept: $y = mx + b$
Point-Slope: $y - y_1 = m(x - x_1)$
General: $Ax + By = C$

Cartesian Space
Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

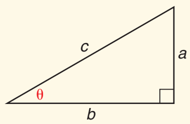
Midpoint: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

Equations of a Line:
Vector Form: $(x, y, z) = (x_1, y_1, z_1) + n(a, b, c)$
Point Form: $(x, y, z) = (x_1 + na, y_1 + nb, z_1 + nc)$

Equation of a Plane:
$Ax + By + Cz = D$

TRIGONOMETRY:

Right Triangle:



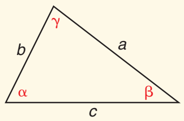
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$

$\sin^2\theta + \cos^2\theta = 1$

Triangle:



$A = \frac{1}{2}bc\sin\alpha$

$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

or

$\cos\gamma = \frac{a^2 + b^2 - c^2}{2ab}$

6E

Elementary Geometry

for College Students

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for College Students

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Elementary Geometry for College Students,
Sixth Edition
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Production Service and Compositor: Graphic World Inc.

Photo Researcher: PreMedia Global

Text Researcher: PreMedia Global

Illustrator: Jade Myers, Matrix Art Services; diacriTech

Text and Cover Designer: Terri Wright

Cover Image: © Sara Meijer/Alamy

Library of Congress Control Number: 2013942407

ISBN-13: 978-1-285-19569-8

ISBN-10: 1-285-19569-8

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Printed in the United States of America
1 2 3 4 5 6 7 17 16 15 14 13

This edition is dedicated to our spouses, children, and grandchildren.
Dan Alexander and Geralyn Koeberlein

LETTER FROM THE AUTHOR

As of the late 1980s, the geometry textbooks written for the elementary college student contained flaws in reasoning as well as errors and even contradictions. In those textbooks, there seemed to be an overall lack of geometric figures that are essential to visual reasoning. Using my collected notes and problems, I developed an outline for the geometry textbook that would present the necessary topics in a logical order. Providing descriptions and explanations that students could read and comprehend, students were able to learn the vocabulary of geometry, recognize visual relationships, solve problems, and even create some proofs of theorems as well. The textbook would have to provide several exercises, many of them serving as building blocks that would transition the student to mid-range and challenging problems and applications. Without doubt, earlier editions of this textbook have evolved so that any improvements would advance my early goals. In time, the Interactive Companion (for added practice and reinforcement) was incorporated into the Student Study Guide. Both the technology for geometry and the applications of geometry are evident throughout this textbook.

As authors, Geralyn Koeberlein and I feel quite strongly that we have always accurately and completely addressed the fundamental concepts of geometry, as suggested by a number of professional mathematical associations. However, many of the changes in the sixth edition are attributable to current users and reviewers of the fifth edition. For instance, we chose to include an increased discussion of the parabola as well as a new section dealing with three-dimensional coordinate geometry. As always, we present new topics concisely and with easily understood explanations.

We continue to include the visual explanations of theorems that are enabled by accurate and well-labeled figures. Comparable to the guidance provided by roadway signs and GPS systems, the geometric figures found in this textbook provide guidance for student readers too. Thus, students will have the tools, figures, and background to "see" results intuitively, explore relationships inductively, and establish principles deductively.

We believe that explanations are a necessary component of the geometry textbook. As well as forcing us to look back (to review), these "proofs" are learning experiences within themselves. Our textbook presents these proofs in the most compact and understandable form that we can find. As the reader will discover, we provide suggestions and insights into the construction of a proof whenever possible.

Writing this textbook for college students, I have incorporated my philosophy for the teaching of geometry. The student who is willing to study geometry and to accept the responsibility and challenges herein will be well-prepared for advanced mathematical endeavors and will also have developed skills of logic that are useful in other disciplines.

Daniel C. Alexander

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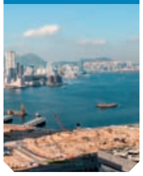

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Preface

As authors, we aim to help the students of the sixth edition of *Elementary Geometry for College Students* become familiar with the terminology of geometry, explore (and perhaps discover) geometric principles, strengthen their skills in deductive reasoning, and gain new skills in problem solving, particularly in the area of geometry-based applications. Our style of writing enables novices of geometric study to open doors, refreshes the memories of students who have had an earlier introduction to the subject, provides a different perspective for other students, and even encourages and directs those who might someday be teaching the subject matter.

As any classroom teacher of geometry would expect, we have developed this textbook in a logical order and with features that are intuitive, informative, and motivational; as a result, students are enabled to achieve the goals cited in the preceding paragraph. For this textbook to be an effective learning tool, it is imperative that it contain a multitude of figures and illustrations; of course, this approach follows from the assumption that "A picture is worth a thousand words."

We are well aware that the completion of a proof is quite the challenge for the student of geometry. With this in mind, we seek initially to have the student recognize and appreciate the role of proof throughout the development of our geometry textbook. To achieve this end, the student will initially be asked to follow the flow of a given proof. In turn, the student should be able to supply missing pieces (statements and reasons) for the proof, thus recognizing a need to both order and justify one's claims. Of course, the ultimate goal is that the student actually generate a geometric proof, beginning with writing a lower-level proof and then progressing toward creating a higher-end proof.

For completeness, convenience, and compactness, we provide proof in a variety of forms: the two-column proof, the paragraph proof, and the "picture" proof. A student's actual creation of the two-column proof demonstrates the student's understanding; that is, it becomes evident that the student can both order and justify conclusions toward a desired end. Our belief is that such accomplishments in the ability to reason extend themselves to other disciplines; for instance, successful students will likely improve paragraph writing in a composition class by improving the order, flow, and even the justification of their claims. Also, the elements of logic found in the study of geometry may very well enable students who are also or will be enrolled in a computer science class to create more powerful and more compact subroutines in their computer codes.

In each edition, we have continued to be inspired and guided by both the National Council of Teachers of Mathematics (NCTM) and the American Mathematical Association of Two-Year Colleges (AMATYC). Of course, we encourage suggestions for content and improvement in this textbook from those who are current users.

OUTCOMES FOR THE STUDENT

- Mastery of the essential concepts of geometry, for intellectual and vocational needs
- Preparation of the transfer student for further study of mathematics and geometry at the senior-level institution
- Understanding of the step-by-step reasoning necessary to fully develop a mathematical system such as geometry
- Enhancement of one's interest in geometry through discovery activities, features, and solutions to exercises

FEATURES OF THE SIXTH EDITION

- Inclusion of approximately 150 new exercises, many of a challenging nature
- Increased uniformity in the steps outlining construction techniques
- Creation of a new Section 10.6 that discusses analytic geometry in three dimensions
- Extension of the feature *Strategy for Proof*, which provides insight into the development of proofs of geometric theorems
- Expanded coverage of parabolas
- Extension of the Discover activities
- Extension of Appendix A.4 of the Fifth Edition—now Appendixes A.4, Factoring and Quadratic Equations, and A.5, The Quadratic Formula and Square Root Properties of the Sixth Edition

TRUSTED FEATURES

Full-color format aids in the development of concepts, solutions, and investigations through application of color to all figures and graphs. The authors have continued the introduction of color to all figures to ensure that it is both accurate and instructionally meaningful.
Reminders found in the text margins provide a convenient recall mechanism.
Discover activities emphasize the importance of induction in the development of geometry.
Geometry in Nature and **Geometry in the Real World** illustrate geometry found in everyday life.
Overviews found in chapter-ending material organize important properties and other information from the chapter.
An **Index of Applications** calls attention to the practical applications of geometry.
A **Glossary of Terms** at the end of the textbook provides a quick reference of geometry terms.
Chapter-opening photographs highlight subject matter for each chapter.
Warnings are provided so that students might avoid common pitfalls.
Chapter Summaries review the chapter, preview the chapter to follow, and provide a list of important concepts found in the current chapter.
Perspective on History boxes provide students with biographical sketches and background leading to geometric discoveries.
Perspective on Applications boxes explore classical applications and proofs.
Chapter Reviews provide numerous practice problems to help solidify student understanding of chapter concepts.
Chapter Tests provide students the opportunity to prepare for exams.
Formula pages at the front of the book list important formulas with relevant art to illustrate.
Reference pages at the back of the book summarize the important abbreviations and symbols used in the textbook.

STUDENT RESOURCES

Student Study Guide with Solutions Manual (978-1-285-19681-7) provides worked-out solutions to select odd-numbered problems from the text as well as new Interactive Exercise sets for additional review. Select solutions for the additional Interactive Exercise sets are provided within the study guide. Complete solutions are available on the instructors website.

Text-Specific DVDs (978-1-285-19687-9), hosted by Dana Mosely, provide professionally produced content that covers key topics of the text, offering a valuable resource to augment classroom instruction or independent study and review.

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Cognero (Included on the Instructor Companion Website) Cengage Learning Testing Powered by Cognero is a flexible, online system that allows you to author, edit, and manage test bank content from multiple Cengage Learning solutions; create multiple test versions in an instant; and deliver tests from your LMS, your classroom, or wherever you want.

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Everything you need for your course in one place! This collection of book-specific lecture and class tools formerly found on the PowerLecture CD, is available online at **www.cengage.com/login**. Here you can access and download PowerPoint presentations, images, and more.

ACKNOWLEDGMENTS

The authors wish to thank Marc Bove, Rita Lombard, and Maureen Ross at Cengage for their efforts in making the sixth edition of the textbook a reality. Also instrumental in the process were Amy Simpson of Graphic World Inc. and Megan Guiney at diacriTech, Inc. In addition, we would like to recognize and thank those who made earlier editions of this textbook possible: Beth Dahlke, Theresa Grutz, Florence Powers, Dawn Nuttall, Lynn Cox, Melissa Parkin, and Noel Kamm.

We express our gratitude to reviewers of the current edition and previous editions:

Houssam M. Al-Basha, *Lincoln Land Community College*
Paul Allen, *University of Alabama*
Jane C. Beatie, *University of South Carolina at Aiken*
Dr. Michaela Bertch, *Cathedral Catholic High School*
Steven Blasberg, *West Valley College*
Barbara Brown, *Anoka Ramsey Community College*
Patricia Clark, *Indiana State University*
Joyce Cutler, *Framingham State College*
Walter Czarnec, *Framingham State College*
Kahroline di Passero, *Moorpark Community College*
Darwin G. Dorn, *University of Wisconsin–Washington County*
William W. Durand, *Henderson State University*
Zoltan Fischer, *Minneapolis Community and Technical College*
Kathryn E. Godshalk, *Cypress College*
Chris Graham, *Mt. San Antonio Community College*
Sharon Gronberg, *Southwest Texas State University*
Geoff Hagopian, *College of the Desert*
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Ben L. Hill, *Lane Community College*
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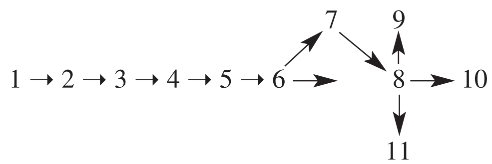
Tracy Hoy, *College of Lake County*
Greg Jamison, *Fresno City College*
Josephine G. Lane, *Eastern Kentucky University*
John C. Longnecker, *University of Northern Iowa*
Erin C. Martin, *Parkland College*
Nicholas Martin, *Shepherd College*
Jill McKenney, *Lane Community College*
James R. McKinney, *Cal Poly at Pomona*
Iris C. McMurtry, *Motlow State Community College*
Jack Morrell, *Atlanta Metropolitan State College*
Michael Naylor, *Western Washington University*
Maurice Ngo, *Chabot College*
Ellen L. Rebold, *Brookdale Community College*
Lauri Semarne, *Los Angeles, California*
Patty Shovanec, *Texas Technical University*
Marvin Stick, *University of Massachusetts–Lowell*
Joseph F. Stokes, *Western Kentucky University*
Kay Stroope, *Phillips Community College–University of Arkansas*
Dr. John Stroyls, *Georgia Southwestern State University*
Karen R. Swick, *Palm Beach Atlantic College*
Steven L. Thomassin, *Ventura College*
Bettie A. Truitt, *Black Hawk College*
Jean A. Vrechek, *Sacramento City College*
Tom Zerger, *Saginaw Valley State University*

Foreword

In the sixth edition of *Elementary Geometry for College Students*, the topics that comprise a minimal course include most of Chapters 1–6 and Chapter 8. For a complete basic course, coverage of Chapters 1–8 is recommended. Some sections that can be treated as optional in formulating a course description include the following:

- Section 2.6 Symmetry and Transformations
- Section 3.4 Basic Constructions Justified
- Section 3.5 Inequalities in a Triangle
- Section 5.6 Segments Divided Proportionally
- Section 6.4 Some Constructions and Inequalities for the Circle
- Section 7.1 Locus of Points
- Section 7.2 Concurrence of Lines
- Section 7.3 More About Regular Polygons
- Section 8.5 More Area Relationships in the Circle
- Section 10.6 The Three-Dimensional Coordinate System

Given that this textbook is utilized for three-, four-, and five-hour courses, the following flowchart depicts possible orders in which the textbook can be used. As suggested by the preceding paragraph, it is possible to treat certain sections as optional.



For students who need further review of related algebraic topics, consider these topics found in Appendix A:

A.1: Algebraic Expressions
A.2: Formulas and Equations
A.3: Inequalities
A.4: Factoring and Quadratic Equations
A.5: The Quadratic Formula and Square Root Properties

Sections A.4 and A.5 include these methods of solving quadratic equations: the factoring method, the square roots method, and the Quadratic Formula.

Logic appendices can be found at the textbook website. These include:

Logic Appendix 1: Truth Tables
Logic Appendix 2: Valid Arguments

Daniel C. Alexander and Geralyn M. Koeberlein

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6E

Elementary Geometry

for College Students

Doug Byrnes/Corbis

Chapter 1 Line and Angle Relationships

CHAPTER OUTLINE

1.1 Sets, Statements, and Reasoning
1.2 Informal Geometry and Measurement
1.3 Early Definitions and Postulates
1.4 Angles and Their Relationships
1.5 Introduction to Geometric Proof
1.6 Relationships: Perpendicular Lines
1.7 The Formal Proof of a Theorem
■ **PERSPECTIVE ON HISTORY:** The Development of Geometry
■ **PERSPECTIVE ON APPLICATIONS:** Patterns
■ **SUMMARY**

Magical! In geometry, figures can be conceived to create an illusion. Known as the Bridge of Aspiration, this passageway was conceptualized by the Wilkinson Eyre Architects. It connects the Royal Opera House and the Royal Ballet School in Covent Garden in London, England. In this geometric design, 23 square portals are each rotated slightly in order to create the illusion of a twisted passage. Although a visual inspection of the bridge might have one think that people would have to walk on walls to cross Floral Street below, it is easy to walk upright. The architectural design successfully creates the fluidity, grace, and spirit of the dance. This chapter opens with a discussion of the types of reasoning: intuition, induction, and deduction. Additional topics found in Chapter 1 and useful for design include tools of geometry, such as the ruler, protractor, and compass. By considering relationships between lines and angles, the remainder of the chapter begins the logical development of geometry. For the geometry student needing an algebra review, several topics are found in Appendix A. Other topics are developed as needed.

Additional video explanations of concepts, sample problems, and applications are available on DVD.

1.1 Sets, Statements, and Reasoning

KEY CONCEPTS

Statement	Conclusion	Law of Detachment
Variable	Reasoning	Set
Conjunction	Intuition	Subset
Disjunction	Induction	Venn Diagram
Negation	Deduction	Intersection
Implication (Conditional)	Argument (Valid and Invalid)	Union
Hypothesis		

SETS

A **set** is any collection of objects, all of which are known as the *elements* of the set. The statement $A = \{1, 2, 3\}$ is read, "A is the set of elements 1, 2, and 3." In geometry, geometric figures such as lines and angles are actually sets of points.

Where $A = \{1, 2, 3\}$ and $B = \{\text{counting numbers}\}$, A is a *subset* of B because each element in A is also in B; in symbols, $A \subseteq B$. In Chapter 2, we will discover that $T = \{\text{all triangles}\}$ is a subset of $P = \{\text{all polygons}\}$; that is, $T \subseteq P$.

STATEMENTS

DEFINITION

A **statement** is a set of words and/or symbols that collectively make a claim that can be classified as true or false.

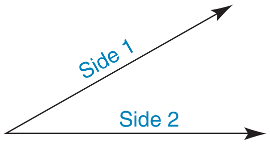


Figure 1.1

EXAMPLE 1

Classify each of the following as a true statement, a false statement, or neither.

1. $4 + 3 = 7$
2. An angle has two sides. (See Figure 1.1.)
3. Robert E. Lee played shortstop for the Yankees.
4. $7 < 3$ (This is read "7 is less than 3.")
5. Look out!

SOLUTION 1 and 2 are true statements; 3 and 4 are false statements; 5 is not a statement.

Some statements contain one or more *variables*; a **variable** is a letter that represents a number. The claim "$x + 5 = 6$" is called an *open sentence* or *open statement* because it can be classified as true or false, depending on the replacement value of x. For instance, $x + 5 = 6$ is true if $x = 1$; for x not equal to 1, $x + 5 = 6$ is false. Some statements containing variables are classified as true because they are true for all replacements. Consider the Commutative Property of Addition, usually stated in the form $a + b = b + a$. In words, this property states that the same result is obtained when two numbers are added in either order; for instance, when $a = 4$ and $b = 7$, it follows that $4 + 7 = 7 + 4$.

The **negation** of a given statement P makes a claim opposite that of the original statement. If the given statement is true, its negation is false, and vice versa. If P is a statement, we use $\sim P$ (which is read "not P") to indicate its negation.

EXAMPLE 2

Give the negation of each statement.

a) $4 + 3 = 7$ **b)** All fish can swim

SOLUTION

a) $4 + 3 \neq 7$ ($\neq$ means "is not equal to.")

b) Some fish cannot swim. (To negate "All fish can swim," we say that at least one fish cannot swim.)

TABLE 1.1
The Conjunction

P	*Q*	*P* and *Q*
T	T	T
T	F	F
F	T	F
F	F	F

TABLE 1.2
The Disjunction

P	*Q*	*P* or *Q*
T	T	T
T	F	T
F	T	T
F	F	F

A *compound* statement is formed by combining other statements used as "building blocks." In such cases, we may use letters such as P and Q to represent simple statements. For example, the letter P may refer to the statement "$4 + 3 = 7$," and the letter Q to the statement "Babe Ruth was a U.S. president." The statement "$4 + 3 = 7$ *and* Babe Ruth was a U.S. president" has the form *P and Q,* and is known as the **conjunction** of P and Q. The statement "$4 + 3 = 7$ *or* Babe Ruth was a U.S. president" has the form *P or Q,* and is known as the **disjunction** of statement P and statement Q. A conjunction is true only when P and Q are *both* true. A disjunction is false only when P and Q are *both* false. See Tables 1.1 and 1.2.

EXAMPLE 3

Assume that statement P and statement Q are both true.

P: $4 + 3 = 7$
Q: An angle has two sides.

Classify the following statements as true or false.

1. $4 + 3 \neq 7$ and an angle has two sides.
2. $4 + 3 \neq 7$ or an angle has two sides.

SOLUTION Statement 1 is false because the conjunction has the form "F and T." Statement 2 is true because the disjunction has the form "F or T."

The statement "If P, then Q," known as a **conditional statement** (or **implication**), is classified as true or false as a whole. A statement of this form can be written in equivalent forms; for instance, the conditional statement, "If an angle is a right angle, then it measures 90 degrees" is equivalent to the statement, "All right angles measure 90 degrees."

EXAMPLE 4

Classify each conditional statement as true or false.

1. If an animal is a fish, then it can swim. (States, "All fish can swim.")
2. If two sides of a triangle are equal in length, then two angles of the triangle are equal in measure. (See Figure 1.2 below.)

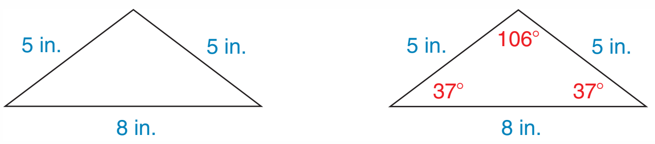


Figure 1.2

3. If Wendell studies, then he will receive an A on the test.

SOLUTION Statements 1 and 2 are true. Statement 3 is false; Wendell may study yet not receive an A.

In the conditional statement "If P, then Q," P is the **hypothesis** and Q is the **conclusion.** In statement 2 of Example 4, we have

Hypothesis: Two sides of a triangle are equal in length.

Conclusion: Two angles of the triangle are equal in measure.

SSG **EXS. 1–7**

For the true statement "If P, then Q," the hypothetical situation described in P implies the conclusion described in Q. This type of statement is often used in reasoning, so we turn our attention to this matter.

REASONING

Success in the study of geometry requires vocabulary development, attention to detail and order, supporting claims, and thinking. **Reasoning** is a process based on experience and principles that allows one to arrive at a conclusion. The following types of reasoning are used to develop mathematical principles.

1.	Intuition	An inspiration leading to the statement of a theory
2.	Induction	An organized effort to test and validate the theory
3.	Deduction	A formal argument that proves the tested theory

Intuition

We are often inspired to think and say, "It occurs to me that. . . ." With **intuition,** a sudden insight allows one to make a statement without applying any formal reasoning. When intuition is used, we sometimes err by "jumping" to conclusions. In a cartoon, the character having the "bright idea" (using intuition) is shown with a light bulb next to her or his head.

EXAMPLE 5

Figure 1.3 is called a *regular pentagon* because its five sides have equal lengths and its five interior angles have equal measures. What do you suspect is true of the lengths of the dashed parts of lines from B to E and from B to D?

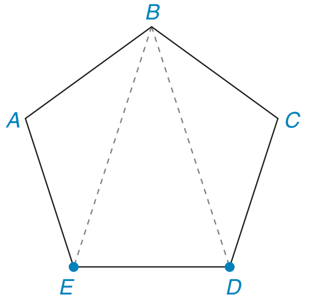


Figure 1.3

SOLUTION Intuition suggests that the lengths of the dashed parts of lines (known as *diagonals* of the pentagon) are the same.

NOTE 1: Using induction (and a *ruler*), we can verify that this claim is true. We will discuss measurement with the ruler in more detail in Section 1.2.

NOTE 2: Using methods found in Chapter 3, we could use deduction to prove that the two diagonals do indeed have the same length.

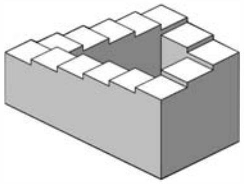
The role intuition plays in formulating mathematical thoughts is truly significant. But to have an idea is not enough! Testing a theory may lead to a revision of the theory or even to its total rejection. If a theory stands up to testing, it moves one step closer to becoming mathematical law.

Induction

We often use specific observations and experiments to draw a general conclusion. This type of reasoning is called **induction.** As you would expect, the observation/experimentation process is common in laboratory and clinical settings. Chemists, physicists, doctors, psychologists,

Discover

An optical illusion known as "Penrose stairs" is shown below. Although common sense correctly concludes that no such stairs can be constructed, what unusual quality appears to be true of the stairs drawn?



ANSWER

The stairs constantly rise or descend.

weather forecasters, and many others use collected data as a basis for drawing conclusions. In our study of geometry, the inductive process generally has us use the ruler or the *protractor* (to measure angles).

EXAMPLE 6

While in a grocery store, you examine several 6-oz cartons of yogurt. Although the flavors and brands differ, each carton is priced at 75 cents. What do you conclude?

CONCLUSION Every 6-oz carton of yogurt in the store costs 75 cents.

As you may already know (see Figure 1.2), a figure with three straight sides is called a *triangle*.

EXAMPLE 7

In a geometry class, you have been asked to measure the three interior angles of each triangle in Figure 1.4. You discover that triangles I, II, and IV have two angles (as marked) that have equal measures. What may you conclude?

CONCLUSION The triangles that have two sides of equal length also have two angles of equal measure.

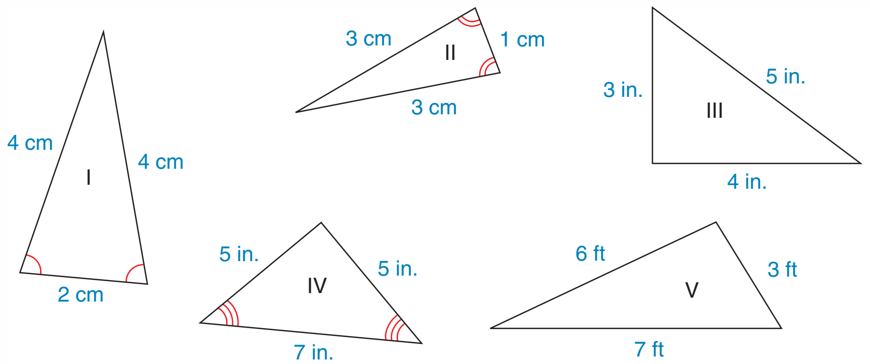


Figure 1.4

NOTE: The protractor, used to support the conclusion above, will be discussed in Section 1.2.

Deduction

DEFINITION

Deduction is the type of reasoning in which the knowledge and acceptance of selected assumptions guarantee the truth of a particular conclusion.

In Example 8, we illustrate a **valid argument,** a form of deductive reasoning used frequently in the development of geometry. In this form, at least two statements are treated as facts; these assumptions are called the *premises* of the argument. On the basis of the premises, a particular *conclusion* must follow. This form of deduction is called the **Law of Detachment.**

EXAMPLE 8

If you accept the following statements 1 and 2 as true, what must you conclude?

1. If a student plays on the Rockville High School boys' varsity basketball team, then he is a talented athlete.
2. Todd plays on the Rockville High School boys' varsity basketball team.

CONCLUSION Todd is a talented athlete.

To more easily recognize this pattern for deductive reasoning, we use letters to represent statements in the following generalization.

LAW OF DETACHMENT

Let P and Q represent simple statements, and assume that statements 1 and 2 are true. Then a valid argument having conclusion C has the form

1. If P, then Q
2. P } **premises**
C. $\therefore Q$ } **conclusion**

NOTE: The symbol $\therefore$ means "therefore."

In the preceding form, the statement "If P, then Q" is often read "P implies Q." That is, when P is known to be true, Q must follow.

EXAMPLE 9

Is the following argument valid? Assume that premises 1 and 2 are true.

1. If it is raining, then Tim will stay in the house.
2. It is raining.
C. $\therefore$ Tim will stay in the house.

CONCLUSION The argument is valid because the form of the argument is

1. If P, then Q
2. P
C. $\therefore Q$

with P = "It is raining," and Q = "Tim will stay in the house."

EXAMPLE 10

Is the following argument valid? Assume that premises 1 and 2 are true.

1. If a man lives in London, then he lives in England.
2. William lives in England.
C. $\therefore$ William lives in London.

CONCLUSION The argument is not valid. Here, P = "A man lives in London," and Q = "A man lives in England." Thus, the form of this argument is

1. If P, then Q
2. Q
C. $\therefore P$

To represent a valid argument, the Law of Detachment would require that the first statement has the form "If Q, then P." Even though statement Q is true, it does not enable us to draw a valid conclusion about P. Of course, if William lives in England, he *might* live in London; but he might instead live in Liverpool, Manchester, Coventry, or any of countless other places in England. Each of these possibilities is a **counterexample** disproving the validity of the argument. Remember that deductive reasoning is concerned with reaching conclusions that *must be true,* given the truth of the premises.

Warning

In the box, the argument on the left is valid and patterned after Example 9. The argument on the right is invalid; this form was given in Example 10.

VALID ARGUMENT	INVALID ARGUMENT
1. If P, then Q	**1.** If P, then Q
2. P	**2.** Q
C. $\therefore Q$	C. $\therefore P$

We will use deductive reasoning throughout our work in geometry. For example, suppose that you know these two facts:

SSG **EXS. 8–12**

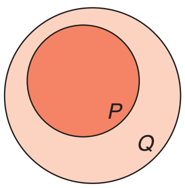
1. If an angle is a right angle, then it measures 90°.
2. Angle A is a right angle.

Because the form found in statements 1 and 2 matches the form of the valid argument, you may draw the following conclusion.

C. Angle A measures 90°.

VENN DIAGRAMS

Sets of objects are often represented by geometric figures known as *Venn Diagrams.* Their creator, John Venn, was an Englishman who lived from 1834 to 1923. In a Venn Diagram, each set is represented by a closed (bounded) figure such as a circle or rectangle. If statements P and Q of the conditional statement "If P, then Q" are represented by sets of objects P and Q, respectively, then the Law of Detachment can be justified by a geometric argument. When a Venn Diagram is used to represent the statement "If P, then Q," it is absolutely necessary that circle P lies in circle Q; that is, P is a *subset* of Q. (See Figure 1.5.)



If P, then Q.

Figure 1.5

EXAMPLE 11

Use Venn Diagrams to verify Example 8.

SOLUTION Let B = students on the Rockville High varsity boys' basketball team. Let A = people who are talented athletes.

To represent the statement "If a basketball player (B), then a talented athlete (A)," we show B within A. In Figure 1.6 we use point T to represent Todd, a person on the basketball team (T in B). With point T also in circle A, we conclude that "Todd is a talented athlete."

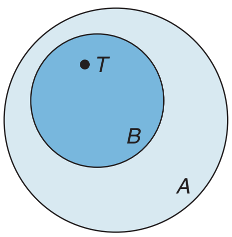


Figure 1.6

The statement "If P, then Q" is sometimes expressed in the form "All P are Q." For instance, the conditional statement of Examples 8 and 11 can be written "All Rockville High School basketball players are talented athletes." Venn Diagrams can also be used to demonstrate that the argument of Example 10 is not valid. To show the invalidity of the argument in Example 10, one must show that an object in Q may *not* lie in circle P. (See Figure 1.5.)

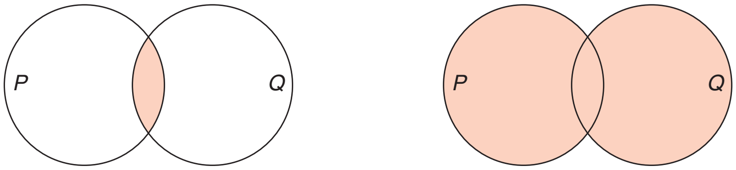
Discover

In the St. Louis area, an interview of 100 sports enthusiasts shows that 74 support the Cardinals baseball team and 58 support the Rams football team. All of those interviewed support one team or the other or both. How many support both teams?

ANSWER
32; 74 + 58 − 100

The compound statements known as the conjunction and the disjunction can also be related to the intersection and union of sets, relationships that can be illustrated by the use of Venn Diagrams. For the Venn Diagram, we assume that the sets P and Q may have elements in common. (See Figure 1.7.)

The elements common to P and Q form the **intersection** of P and Q, which is written $P \cap Q$. This set, $P \cap Q$, is the set of all elements in *both* P **and** Q. The elements that are in P, in Q, or in both form the **union** of P and Q, which is written $P \cup Q$. This set, $P \cup Q$, is the set of elements in P **or** Q.



(a) $P \cap Q$ (b) $P \cup Q$

Figure 1.7

SSG **EXS. 13–15**

Exercises 1.1

In Exercises 1 and 2, which sentences are statements? If a sentence is a statement, classify it as true or false.

1. a) Where do you live?
b) $4 + 7 \neq 5$.
c) Washington was the first U.S. president.
d) $x + 3 = 7$ when $x = 5$.

2. a) Chicago is located in the state of Illinois.
b) Get out of here!
c) $x < 6$ (read as "x is less than 6") when $x = 10$.
d) Babe Ruth is remembered as a great football player.

In Exercises 3 and 4, give the negation of each statement.

3. a) Christopher Columbus crossed the Atlantic Ocean.
b) All jokes are funny.

4. a) No one likes me.
b) Angle 1 is a right angle.

In Exercises 5 to 10, classify each statement as simple, conditional, a conjunction, or a disjunction.

5. If Alice plays, the volleyball team will win.

6. Alice played and the team won.

7. The first-place trophy is beautiful.

8. An integer is odd or it is even.

9. Matthew is playing shortstop.

10. You will be in trouble if you don't change your ways.

In Exercises 11 to 18, state the hypothesis and the conclusion of each statement.

11. If you go to the game, then you will have a great time.

12. If two chords of a circle have equal lengths, then the arcs of the chords are congruent.

13. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

14. If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $a \cdot d = b \cdot c$.

15. Corresponding angles are congruent if two parallel lines are cut by a transversal.

16. Vertical angles are congruent when two lines intersect.

17. All squares are rectangles.

18. Base angles of an isosceles triangle are congruent.

In Exercises 19 to 24, classify each statement as true or false.

19. If a number is divisible by 6, then it is divisible by 3.

20. Rain is wet and snow is cold.

21. Rain is wet or snow is cold.

22. If Jim lives in Idaho, then he lives in Boise.

23. Triangles are round or circles are square.

24. Triangles are square or circles are round.

In Exercises 25 to 32, name the type of reasoning (if any) used.

25. While participating in an Easter egg hunt, Sarah notices that each of the seven eggs she has found is numbered. Sarah concludes that all eggs used for the hunt are numbered.

26. You walk into your geometry class, look at the teacher, and conclude that you will have a quiz today.

27. Albert knows the rule "If a number is added to each side of an equation, then the new equation has the same solution set as the given equation." Given the equation $x - 5 = 7$, Albert concludes that $x = 12$.

28. You believe that "Anyone who plays major league baseball is a talented athlete." Knowing that Duane Gibson has just been called up to the major leagues, you conclude that Duane Gibson is a talented athlete.

29. As a handcuffed man is brought into the police station, you glance at him and say to your friend, "That fellow looks guilty to me."

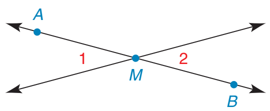
30. While judging a science fair project, Mr. Cange finds that each of the first 5 projects is outstanding and concludes that all 10 will be outstanding.

31. You know the rule "If a person lives in the Santa Rosa Junior College district, then he or she will receive a tuition break at Santa Rosa." Emma tells you that she has received a tuition break. You conclude that she resides in the Santa Rosa Junior College district.

32. As Mrs. Gibson enters the doctor's waiting room, she concludes that it will be a long wait.

In Exercises 33 to 36, use intuition to state a conclusion.

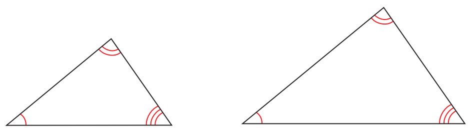
33. You are told that the opposite angles formed when two lines cross are **vertical angles**. In the figure, angles 1 and 2 are vertical angles. Conclusion?



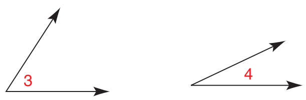
Exercises 33, 34

34. In the figure, point M is called the **midpoint** of line segment AB. Conclusion?

35. The two triangles shown are **similar** to each other. Conclusion?



36. Observe (but do not measure) the following angles. Conclusion?



In Exercises 37 to 40, use induction to state a conclusion.

37. Several movies directed by Lawrence Garrison have won Academy Awards, and many others have received nominations. His latest work, *A Prisoner of Society*, is to be released next week. Conclusion?

38. On Monday, Matt says to you, "Andy hit his little sister at school today." On Tuesday, Matt informs you, "Andy threw his math book into the wastebasket during class." On Wednesday, Matt tells you, "Because Andy was throwing peas in the school cafeteria, he was sent to the principal's office." Conclusion?

39. While searching for a classroom, Tom stopped at an instructor's office to ask directions. On the office bookshelves are books titled *Intermediate Algebra, Calculus, Modern Geometry, Linear Algebra,* and *Differential Equations*. Conclusion?

40. At a friend's house, you see several food items, including apples, pears, grapes, oranges, and bananas. Conclusion?

In Exercises 41 to 50, use deduction to state a conclusion, if possible.

41. If the sum of the measures of two angles is 90°, then these angles are called "complementary." Angle 1 measures 27° and angle 2 measures 63°. Conclusion?

42. If a person attends college, then he or she will be a success in life. Kathy Jones attends Dade County Community College. Conclusion?

43. All mathematics teachers have a strange sense of humor. Alex is a mathematics teacher. Conclusion?

44. All mathematics teachers have a strange sense of humor. Alex has a strange sense of humor. Conclusion?

45. If Stewart Powers is elected president, then every family will have an automobile. Every family has an automobile. Conclusion?

46. If Tabby is meowing, then she is hungry. Tabby is hungry. Conclusion?

47. If a person is involved in politics, then that person will be in the public eye. June Jesse has been elected to the Missouri state senate. Conclusion?

48. If a student is enrolled in a literature course, then he or she will work very hard. Bram Spiegel digs ditches by hand six days a week. Conclusion?

49. If a person is rich and famous, then he or she is happy. Marilyn is wealthy and well known. Conclusion?

50. If you study hard and hire a tutor, then you will make an A in this course. You make an A in this course. Conclusion?

In Exercises 51 to 54, use Venn Diagrams to determine whether the argument is valid or not valid.

51.
1) If an animal is a cat, then it makes a "meow" sound.
2) Tipper is a cat.
C) Then Tipper makes a "meow" sound.

52.
1) If an animal is a cat, then it makes a "meow" sound.
2) Tipper makes a "meow" sound.
C) Then Tipper is a cat.

53.
1) All Boy Scouts serve the United States of America.
2) Sean serves the United States of America.
C) Sean is a Boy Scout.

54.
1) All Boy Scouts serve the United States of America.
2) Sean is a Boy Scout.
C) Sean serves the United States of America.

55. Where $A = \{1,2,3\}$ and $B = \{2,4,6,8\}$, classify each of the following as true or false.
a) $A \cap B = \{2\}$
b) $A \cup B = \{1,2,3,4,6,8\}$
c) $A \subseteq B$